Optimal Energy Extraction During Dynamic Jet Stream Soaring

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Abstract

Dynamic soaring, a technique in which horizontal wind that varies in strength or direction is used to support flight, could potentially support perpetual flight of a high performance glider in the jet stream. However, the aircraft's control systems would still require electric power. This could be extracted for example using specially designed ram air turbines (RAT). The present paper focuses on developing flight trajectories that extract the maximum amount of power for a RAT equipped glider during dynamic soaring in high altitude jet streams. The physics are worked out, a numerical scheme developed, and preliminary results presented.

Introduction

Lord Rayleigh [1883] is usually accredited for first suggesting that soaring can be done in a horizontal but non-uniform wind field. Seabirds like albatrosses are known to travel hundreds of kilometers in a single day utilizing dynamic soaring (e.g., Cone [1964]). Save for an anecdote of a flight in Australia in 1974, attempts for sustained dynamic soaring of manned gliders appear to not have been fully successful as of yet. However, radio controlled hobby aircraft routinely dynamically soar in the steep wind gradients on the leeward side of mountain ridges and often reach speeds over 600 km/h. A number of papers have been devoted to optimizing flight trajectories of dynamically soaring aircraft, flying either in near-ground wind gradients or in wind gradients associated with high altitude jet streams; e.g. Boslough [2002], Zhao [2004], Sachs [2006], Gordon [2006].

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order to sustain flight over long periods of time, such as weeks, months or years, onboard systems including controls, navigation, transponders, etc, would need to be supplied with power. Power could be generated for example from solar cells or ram air turbines. The focus of the present paper is on the latter.

**Actuator Disk Theory Estimate of Ram Air Turbine Performance**

Classical actuator disk theory is often used for estimating performance of wind turbines and propellers. The theory can be used also for estimating power generation and drag additions of a ram air turbine (RAT). At present only the results are presented:

\[
P_{\text{rat}} = \frac{\eta_{\text{rat}} \rho V^3 A_{\text{rat}} c_1}{4}, \quad c_1 = \left(1 + \frac{V_2}{V} - \left(\frac{V_2}{V}\right)^2 - \left(\frac{V_2}{V}\right)^3\right)
\]

\[
D_{\text{rat}} = \frac{\rho V^2 A_{\text{rat}} c_2}{2}, \quad c_2 = \left(1 - \left(\frac{V_2}{V}\right)^2\right)
\]

where \(P_{\text{rat}}\) is the electric power generated by the RAT, \(A_{\text{rat}}\) is the RAT’s turbine disk area, \(\eta_{\text{rat}}\) is the efficiency of the RAT, \(\rho\) is air density, \(V\) is air speed, \(D_{\text{rat}}\) is the additional aerodynamic drag due to the RAT, and \(c_1\) and \(c_2\) are functions depending on exit velocity \(V_2\) of the air from the RAT. For the present analyses \(c_1\) and \(c_2\) will be assumed to be constant. In the ideal actuator disk theory \(\eta_{\text{rat}}=1\); a more realistic number may be \(\eta_{\text{rat}}=0.5\) or even less. For the trajectory optimizations, \(A_{\text{rat}}\) will be assumed to be a continuous control parameter with which the RAT can be "throttled." For a closed RAT, \(A_{\text{rat}}=0\) resulting in no power generation, \(P_{\text{rat}}=0\), and no additional drag, \(D_{\text{rat}}=0\). For a small UAV flying in the in the jet stream, on the order of 10 W may be required for navigation, controls, transponder, etc. Assuming the air speed to be on the order of \(V=80\) m/s at \(h=11,000\) m altitude, where according to the International Standard Atmosphere (ISA) the air density is \(\rho=0.365\) kg/m\(^3\), and assuming \(V_2/V=0.7\) and \(\eta_{\text{rat}}=0.5\), a turbine area of...
$A_{rat}=4.94 \text{ cm}^2$ would be required for generating $P_{rat}=10 \text{ W}$. The additional drag area would be $D_{rat}/q=2.52 \text{ cm}^2$, where $q=\rho V^2/2$ is dynamic pressure.

**Equations of Motion of Dynamically Soaring Aircraft**

Assume that the aircraft track relative to the air is along the unit vector

$$
e^V = \cos \gamma \sin \psi \hat{e}_1 + \cos \gamma \cos \psi \hat{e}_2 + \sin \gamma \hat{e}_3$$

(2)

where $\psi$ is the heading angle measured clockwise from North, $\gamma$ is air-relative flight path angle, and $\hat{e}_1$ (pointing East), $\hat{e}_2$ (pointing North) and $\hat{e}_3$ (pointing up) are unit vectors in an Earth fixed Cartesian reference frame. The flight trajectory follows the air relative track plus the wind drift,

$$\dot{\mathbf{r}} = \dot{x} \hat{e}_1 + \dot{y} \hat{e}_2 + \dot{h} \hat{e}_3 = V \hat{e}^V + \mathbf{W} = (V \cos \gamma \sin \psi + W_x) \hat{e}_1 + V \cos \gamma \cos \psi \hat{e}_2 + V \sin \gamma \hat{e}_3$$

(3)

where $x$, $y$, $h$ are Cartesian coordinates, a dot represents time derivation, and the last equality holds when the wind $\mathbf{W}$ only has an $x$-component, $W_x(h)$ that depends only on the altitude $h$. These are the same as eqs. (9-11) in Zhao [2004].

Introducing the bank angle $\mu$, and aerodynamic drag parallel and lift perpendicular to the relative wind of the aircraft, into the point mass equations of motion leads to

$$m \dot{V} = -D - mg \sin \gamma - m \dot{W}_x \cos \gamma \sin \psi$$

$$m V \cos \gamma \dot{\psi} = L \sin \mu - m \dot{W}_x \cos \psi$$

$$m V \dot{\gamma} = L \cos \mu - m g \cos \gamma + m \dot{W}_x \sin \gamma \sin \psi$$

(4)

which are identical to eqs. (6-8) in Zhao [2004].
Optimal Trajectories for Energy Extraction during Dynamic Soaring

It will be assumed that a dynamically soaring aircraft has a small ram air turbine (RAT) that can be "throttled" continuously during flight. The task presently considered is to develop an optimal periodic flight trajectory and RAT control strategy such that the maximum amount of energy can be extracted without the airplane loosing altitude. Further constraints on aircraft long term traveling direction and speed, including loitering (no travel), may be imposed but were presently ignored.

If the wind gradient is constant, i.e.

$$\frac{\partial W}{\partial h} = \beta$$

(5)

where $\beta$ is a constant, then for the general flight where long term travel is disregarded the equations of motion that need to be solved are

$$\frac{\dot{V}}{m} = -\frac{D}{m} - g \sin \gamma - \beta V \sin \gamma \cos \gamma \sin \psi$$

$$\dot{\psi} = \frac{L \sin \mu}{m V \cos \gamma} - \beta \tan \gamma \cos \psi$$

(6)

$$\dot{\gamma} = \frac{L \cos \mu}{m V} - \frac{g \cos \gamma}{V} + \beta \sin^2 \gamma \sin \psi$$

$$\dot{h} = V \sin \gamma$$

Note the $x$ and $y$ coordinates are not needed and thus omitted; if desired they can be obtained afterwards by integration. Lift $L$ and drag $D$ were assumed to be
where $S$ is wing planform area, $C_L$ is coefficient of lift, $D_0$ is zero lift (parasitic drag), $f_0$ is the aircraft's zero lift drag area when the RAT is turned off, $D_i$ is induced drag, $b$ is wing span, and $e$ is Oswald's span efficiency factor. With $t_f$ being the time to complete one cycle, the optimization problem can be formulated as

\[
\text{maximize} \quad \frac{1}{t_f} \int_0^{t_f} P_{rat} \, dt
\]

where our objective is to maximize the average power generated by the RAT over a trajectory cycle subject to the equations of motion, eq. (6), the initial conditions

\[
\gamma(0) = 0 \\
h(0) = h_0
\]  

the constraints

\[
h \geq h_0 \\
C_{L_{\min}} \leq C_L \leq C_{L_{\max}} \\
-\mu_{\max} \leq \mu \leq \mu_{\max} \\
-n_{neg} \leq \frac{L}{mg} \leq n_{pos}
\]

and the terminal conditions
\[V(t_f) = V(0)\]
\[\psi(t_f) = \psi(0) + 2\pi\]
\[\gamma(t_f) = \gamma(0)\]
\[h(t_f) \geq h(0)\]  

(11)

In other words, at the end of the cycle the aircraft should have the same heading with the same attitude and airspeed as at the beginning of the cycle. However, it is free to gain altitude.

**Discretization and Numerical Solution**

As the optimization did not lend itself to an analytical solution, a discrete approximation to the problem was used instead. The time for one cycle, \(t_f\), was discretized as

\[t_k = \frac{k - 1}{N} t_f, \quad k = 1, 2, ..., N\]  

(12)

The optimization variable of the discretized version of eq. (8) can then be written by concatenating the aircraft state variables \(x_k = [V_k, \psi_k, \gamma_k, h_k]^T\) and the control inputs \(u_k = [C_{L,k}, \mu_k, A_{rat,k}]^T\) at each time step with the cycle time, or equivalently

\[X = [V_1, \psi_1, \gamma_1, h_1, C_{L,1}, \mu_1, A_{rat,1}, ..., V_N, \psi_N, \gamma_N, h_N, C_{L,N}, \mu_N, A_{rat,N}, t_f]^T\]  

(13)

which has dimension \(7N+1\).

To ensure the equations of motion in eq. (6) are obeyed, they were modeled as non-linear equality constraints in the optimization problem. The equations of motion can be written collectively as
\[ \dot{x} = f(x, u) \]  

Assume that all functions \( x \) are continuous, continuously differentiable, and cubic within each time interval. Since \( \dot{x} = f \), the functions \( f \) can be assumed to be quadratic within each time interval. Then \( x_{k+1} \) can be "exactly" calculated from \( x_k \),

\[
x_{k+1} = x_k + \int_{t_k}^{t_{k+1}} \dot{x} \, dt = x_k + \int_{t_k}^{t_{k+1}} f \, dt = \{\text{since } f \text{ is quadratic} \} = x_k + \frac{t_{k+1} - t_k}{6} \left( f_{k+1} + 4f_{m,k} + f_k \right) \tag{15}
\]

where \( f_{m,k} \) is the value of \( f \) at the midpoint between \( t_k \) and \( t_{k+1} \). The midpoint value of \( f \) can be obtained by inserting midpoint values of \( x \) and \( u \) into the known function \( f \). The midpoint value \( x_{m,k} \) is obtained from the fact that \( x \) was assumed to be cubic and from the endpoint values \( x_k, x_{k+1}, \dot{x}_k = f_k, \dot{x}_{k+1} = f_{k+1} \);

\[
x_{m,k} = \frac{1}{2} \left( x_{k+1} + x_k \right) + \frac{t_{k+1} - t_k}{8} \left( f_k - f_{k+1} \right) \tag{16}
\]

If the control \( u \) is assumed to be linear within each time interval, then the midpoint value of the control is simply

\[
u_{m,k} = \frac{1}{2} \left( u_{k+1} + u_k \right) \tag{17}
\]

Thus the motion constraints can be expressed solely in terms of the optimization variables at each time-step. The objective function is

\[
\frac{1}{t_f} \int_0^{t_f} P_{ra} \, dt = \frac{\eta_{me} c_1}{4 t_f} \int_0^{t_f} \rho V^3 A_{ra} \, dt \tag{18}
\]
where the air density was a function of altitude according to the International Standard Atmosphere (ISA) was used,

\[
\rho = \begin{cases} 
\rho_0 \left( \frac{T_0 - Lh}{T_0} \right)^{\frac{g}{RT}} & , \ h \leq h_s \\
\rho_s e^{-\frac{g}{RT}(h-h_s)} & , \ h > h_s 
\end{cases}
\]

\( (19) \)

where \( \rho_0 = 1.225 \text{ kg/m}^3 \), \( T_0 = 288.16 \text{ K} \), the lapse rate is \( L = 0.0065 \text{ K/m} \), \( g = 9.80665 \text{ ms}^{-2} \), \( R = 287.1 \text{ m}^2\text{s}^{-2}\text{K}^{-1} \), and \( \rho_s \) and \( T_s = T_0 - Lh_s \) are the values of \( \rho \) and \( T \) at the altitude \( h = h_s = 11,000 \text{ m} \). Note that the objective function could be integrated "exactly" if \( V \) and \( A_{ref} \) were, as assumed above, cubic and linear, respectively, within each time interval. However, the expression is rather lengthy and at present a simple trapezoidal integration was used.

Simulation Results

To further investigate the feasibility of our approach, simulations were conducted where optimized aircraft trajectories were automatically generated. The glider model used had a wingspan of 6.0 meters, a wing area of 1.8 \( \text{m}^2 \), a mass of 20 kg, a RAT area of 4.94 \( \text{cm}^2 \), and a RAT efficiency of 0.5. The initial aircraft altitude \( h_0 = 10,000 \text{ meters} \), and the wind gradient was assumed constant at 0.08 \( \text{s}^{-1} \). The time interval was discretized with \( N = 31, 61, \) or 121. A reasonable compromise between accuracy and computational speed was \( N = 61 \). Representative results are shown at Figs. 1-2. The average power generated by the RAT over the trajectory in these figures was 5.94 watts.

The optimized trajectory is shown below qualitatively similar to conventional dynamic soaring trajectories with no energy generation. At the bottom of the trajectory there is an almost horizontal high speed 180\(^\circ\) turn into the wind, followed by a climb into the wind.
Near the top of the trajectory the aircraft almost does a half inside loop. It then dives down with the wind and starts over.

For the trajectory shown in Fig. 1, the optimized RAT control was to keep it fully turned on along the whole flight. Preliminary results indicate that for weaker wind gradients (smaller values of $W_x$), the optimal strategy is to throttle the RAT during the flight rather than to keep it turned on fully.

**Fig. 1.** Flight trajectory obtained by maximizing (average) power generation.
**Fig. 2.** Velocity for the flight trajectory shown above (plotted versus time increment rather than natural time).

**Discussions**

A numerical scheme for optimizing flight trajectories such that a dynamically soaring airplane can extract the maximum amount of energy was developed. The wind gradient was assumed to be constant, varying neither with position nor time, and known. These are severe restrictions. It is expected that extensive research will be needed to developed procedures to estimate the wind fields aloft in order to sustain a dynamically soaring UAV for any appreciable time. The authors have developed a smaller 3.5 m span UAV with an onboard autopilot with GPS, inertial measurement unit, etc. The next challenge is to develop the hardware and software to map the wind, and from the wind map quickly generate optimal flight trajectories and RAT control strategies for the soaring aircraft. A second challenge is to develop an efficient RAT. Fig. 3 shows two CNC turned wind tunnel RAT models presently being studied.
Fig. 3. Two wind tunnel ram air turbine (RAT) models presently studied.

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